GEC223: FLUID MECHANICS

MODULE 4: HYDROPOWER SYSTEMS

TOPIC: REACTION TURBINES-FRANCIS TURBINE

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In reaction turbines, the runner utilizes both potential and kinetic energies to generate power. As the water flows through the moving parts, there is a change both in pressure and in direction and velocity of flow of water.

For modern Francis turbine shown above which is essentially an inward mixed flow reaction turbine, water under pressure, enters the runner from the guide vanes towards the centre in radial direction and discharges out of the runner axially. The head acting on the turbine is partly transformed into kinetic energy (K.E.) and the rest remains as pressure head. The difference in pressure between the guide vanes and the runner called reaction pressure is responsible for the motion of the runner. That is why a Francis turbine is called a reaction turbine.

The moment of the runner is affected by the change in both the potential and kinetic energies of water. After work is done by the water, it is discharged to the tail race by the draft tube, from where the water is discharged outwards from the power house.

WORK DONE AND EFFICIENCY OF FRANCIS TURBINE

NOTE: R_1 = External Radius of turbine wheel

R_2 = Internal Radius of turbine wheel

V_{w1} = Whirl velocity at inlet

V_{w2} = Whirl velocity at outlet

V_1 , V_2 = Absolute velocity at inlet and outlet respectively

V_{f1}, V_{f2} = Flow velocity at inlet and outlet respectively

u_1, u_2 = Tangential velocity at inlet and outlet respectively

V_{r1}, V_{r2} = Relative velocity at inlet and outlet respectively

$\propto$ = Guide vane angle at inlet

θ = Runner vane angle at inlet

$\emptyset$ = Runner vane angle at exit

β = Blade angle at exit

Work done = $\rho Q(V_{w1}u_1 + V_{w2}u_2)$

Recall, w = ρg. This implies, $\rho = \frac{w}{g}$

$\therefore$ Work done = $\frac{wQ}{g}(V_{w1}u_1 + V_{w2}u_2)$ ________________________________Equation 1

Maximum work done or output is obtained when V_{w2} = 0

$\therefore$ Work done = $\frac{wQ}{g}(V_{w1}u_1)$____Equation 2

Eqn 2 occurs when absolute velocity at exit is radial at which β = 90^0.

Hydraulic Efficiency, η_h

Let Net head on a turbine = H where H = H_g - H_f

Where H_g= gross head = difference of water levels between head race and tail race and

h_f= loss of head in penstock

Note, H = Operation head or working head or available head

$\therefore$ Input to the turbine = Wqh

$$\eta_h = \frac{Power\ developed\ by\ the\ runner}{Power\ supplied\ to\ the\ runner = Water\ power}$$

$$\eta_h = \frac{\frac{wQ}{g}(V_{w1}u_1 \pm V_{w2}u_2)}{wQH}$$

$\eta_h = \frac{(V_{w1}u_1 \pm V_{w2}u_2)}{gH}$ ____Equation 3

When the velocity of whirl, V_{w2} at exit is zero,

$\eta_h = \frac{V_{w1}u_1}{gH}$ ____Equation 4

Mechanical Efficiency, η_m

$\eta_m = \frac{Shaft\ Power\ P}{Power\ developed\ by\ the\ runner}$ ____Equation 5

Overall Efficiency, $\eta_o = \frac{Shaft\ water\ power}{Water\ power = Power\ supplied\ to\ the\ turbine}$

$\eta_o = \frac{P}{wQH}$ ____Equation 6

$\eta_o = \eta_h \times \eta_m$ ____Equation 7

η_o varies from 80% to 90%.

Example 1

An inward flow reaction turbine has external and internal diameters a s1.08m and 0.54m. The turbine is running at 200rpm. The width of the turbine at inlet is 240mm and velocity of flow through the runner is constant and is equal to 2.16m/s. The guide blades make an angle of 10^0 to the tangent of the wheel and discharge at the outlet of the turbine is radial. Draw the inlet and outlet velocity triangles and determine:

i. The absolute velocity of water at inlet of the runner
ii. The velocity of whirl at inlet
iii. The relative velocity at inlet
iv. The runner blade angles
v. Width of runner at outlet
vi. Weight of water flowing through the runner per second
vii. Head at inlet of the turbine
viii. Power developed
ix. Hydraulic efficiency of the turbine
x. Specific speed of the turbine if the centre-line of the spiral casing inlet of the turbine is 2.5m above the tail water level.

Solution

External Diameter, D_1 = 1.08m

Internal Diameter, D_2 = 0.54m

Speed, N = 200rpm

Width at inlet, B_1= 240mm = 0.24m

Velocity of flow, V_{f1}= V_{f1}= 2.16m/s

Guide blade angle, ∝ = 10^0.

Since discharge at outlet is radial, β = 90^0 and V_{w2} = 0.

Tangential velocity of wheel at inlet, $u_1 = \frac{\pi D_1 N}{60}$

$\therefore u_1 = \frac{\pi X\ 1.08\ x\ 200}{600}$ = 1.31m/s

Tangential velocity of wheel at outlet, $u_2 = \frac{\pi D_2 N}{60}$

$\therefore u_2 = \frac{\pi X\ 0.54\ x\ 200}{600}$ = 5.65m/s

i. Absolute velocity of water at inlet of runner, V_1

From the inlet velocity triangle, Sin $\alpha = \frac{V_{f1}}{V_1}$

$\therefore V_1 = \frac{V_{f1}}{\text{Sin } \alpha} = \frac{2.16}{Sin\ 10^0}$ = **12.44m/s**

ii. Velocity of whirl at inlet, V_{w1}

Cos $\alpha = \frac{V_{w1}}{V_1}$

$\therefore V_{w1} = V_1$ Cos α = 12.44 x Cos 10^0 = **12.25m/s**

iii. Relative velocity at inlet, V_{r1}

$V_{r1}{}^2 = V_{f1}{}^2 + (V_{w1} - u_1)^2$

$\therefore V_{r1} = \sqrt{V_{f1}{}^2 + (V_{w1} - u_1)^2} = \sqrt{2.16^2 + (12.25 - 11.31)^2}$ = **2.35m/s**

iv. Runner blade angles θ, Ø

From inlet velocity triangles,

tan $\theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{2.16}{12.25 - 11.31}$ =2.298

$\therefore \theta = tan^{-1} 2.298$ = **66.48^0**

From outlet velocity triangle,

$\tan\emptyset = \frac{V_{f2}}{U_2} = \frac{2.16}{5.65} = 0.382$

$\therefore \emptyset = \tan^{-1} 0.382 =$ **20.9^0**

v. Width of runner at outlet, B_2

From continuity equation, $Q_1 = Q_2$

Recall, Q = AV where A = πDB

Where D = diameter of runner vane and B = width of runner vane

$\pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$ Since $V_{f1} = V_{f2}$ and cancels out along with π

$D_1 B_1 = D_2 B_2$

$\therefore B_2 = \frac{D_1 B_1}{D_2} = \frac{1.08 \ x \ 0.24}{0.54} =$ **0.48m = 480mm**.

vi. Weight of water flowing through the runner per second, W = wQ = w x $\pi D_1 B_1 V_{f1}$

W = 9.81 x π x 1.08 x 0.24 x 2.16 = **17.25KN/s**

vii. Head at inlet of turbine, $H_{inlet} = \frac{1}{g}(V_{w1}u_1 + V_{w2}u_2) + \frac{V_2^{\,2}}{2g}$

Since $V_{w2} = 0$,

$H_{inlet} = \frac{1}{g}(V_{w1}u_1) + \frac{V_2^{\,2}}{2g} = \frac{1}{9.81}(12.25 \ x \ 11.31) + \frac{2.16^2}{2 \ x \ 9.81}$

$H_{inlet} =$ **14.36m**

viii. Power developed, P = ρQ X $V_{w1}u_1 = \frac{wQ}{g}$ x $V_{w1}u_1 = \frac{17.25}{9.81}$ x 12.25 x 11.31 = **243.6KW**

ix. Hydraulic Efficiency, $\eta_h = \frac{V_{w1}u_1}{gH} = \frac{12.25 \ x \ 11.31}{9.81 \ x \ 14.36} = 0.9835 =$ **98.35%**

x. Specific Speed of turbine, $N_s = \frac{N\sqrt{P}}{H^{5/4}}$ ________________________________Equation 8

where H = $\frac{P}{w} + \frac{V_2^{\,2}}{2g} + Z$ ____Equation 9

Specific speed of turbine, $N_s = \frac{200\sqrt{243.6}}{16.86^{5/4}} =$ **91.37**

WORKING PROPORTIONS OF A FRANCIS TURBINE

Ratio of width to diameter, $\frac{B}{D}$

The ratio of width, B_1, to the diameter of wheel D_1 at inlet is represented as n where

n = $\frac{B_1}{D_1}$. The value of n varies from 0.10 to 0.45.

Flow ratio, K_f, is the ratio of the velocity of flow at inlet to the theoretical velocity.

$K_f = \frac{V_{f1}}{\sqrt{2gH}}$. The value of K_f varies from 0.15 to 0.30.

Speed ratio, K_u, is the ratio of the peripheral speed at inlet to the theoretical jet velocity

$K_u = \frac{u}{\sqrt{2gH}}$. The value of K_u varies from 0.6 to 0.9.

DESIGN OF A FRANCIS TURBINE RUNNER

Let B_1, D_1 and t_1 respectively be the width, diameter and thickness of runner vane at inlet.

Then total area at the periphery (i.e. at the runner inlet), A = $(\pi D_1 - Z_{t1})B_1 = K_{t1}\pi D_1 B_1$

Where K_{t1} is known as vane thickness factor or coefficient and its value is always less than unity usually between 0.95 and 1.0.

Recall, Discharge Q = Area of flow x Velocity of flow

Q = $K_{t1}\pi D_1 B_1 \times V_{f1}$

∴ Flow velocity, $V_{f1} = \frac{Q}{K_{t1}\pi D_1 B_1}$

Recall, $B_1 = nD_1$

∴ $V_{f1} = \frac{Q}{K_{t1}\pi n D_1^{\,2}}$

Also, $V_{f1} = K_f\sqrt{2gH}$

Equating the two values of V_{f1} gives,

$K_f\sqrt{2gH} = \frac{Q}{K_{t1}\pi n D_1^{\,2}}$

∴ $D_1 = \left[\frac{Q}{(K_f\sqrt{2gH})K_t\pi n}\right]^{1/2}$

$\therefore B_1 = nD_1$

Tangential velocity u_1 is also called rim velocity where $u_1 = \frac{\pi D_1 N}{60}$

Velocity of whirl at inlet V_{w1} derived from $\eta_h = \frac{V_{w1} u_1}{gH}$

$V_{w1} = \frac{\eta_h gH}{u_1}$

$\tan\alpha = \frac{V_{f1}}{V_{w1}}$ and $\tan\theta = \frac{V_{f1}}{V_{w1} - u_1}$ from inlet velocity triangle.

Assuming diameter D_2 at outlet is one-half diameter at inlet,

$\therefore D_2 = \frac{D_1}{2}$ and $u_2 = \frac{u_1}{2}$

Velocity of flow at exit, V_{f2} is obtained as follows:

Q = $K_{t1}\pi D_1 B_1 V_{f1} = K_{t2}\pi D_2 B_2 V_{f2}$ from continuity equation.

$\therefore \frac{V_{f1}}{V_{f2}} = \frac{K_{t2}\pi D_2 B_2}{K_{t1}\pi D_1 B_1}$

Assuming $V_{f1} = V_{f2}$

This implies $B_2 = 2B_1$

Number of vanes from 16 to 24. To avoid periodic impulse, the number of vanes should be either one or more less than the number of guide vanes.

The design of Francis Turbine runner is summarized as follows:

1. Assume suitable values of η_0, η_h, η, K_f and K_t.
2. Determine the required discharge Q from P = η_0 x wQH
3. Obtain the velocity of flow from the discharge and flow area.

ADVANTAGES OF FRANCIS TURBINE OVER PELTON WHEEL

1. In Francis turbine, the variation in the operating head can be more easily controlled.
2. The operating head can be utilized even when the variation in the tail water level is relatively large when compared to the total head.
3. The mechanical efficiency of Pelton wheel decreases faster with wear than Francis turbine.
4. The size of the runner, generator and power house required for Francis turbine is small and economical compared to Pelton wheel for the same power generation.

DISADVANTAGES OF FRANCIS TURBINE COMPARED TO PELTON WHEEL

1. Water which is not clean can very rapid wear in high head Francis turbine.
2. The overhaul and inspection is much more difficult comparatively.
3. Cavitaion is an ever-present danger.
4. The water hammer effect is more troublesome with Francis turbine.

Example 2

The following data pertain to an inward flow reaction turbine:

Net head____86.4m

Speed of runner______________________________________650rpm

Shaft power available_________________________________ 397KW

Ratio of wheel width to wheel diameter at inlet______________0.10

Ratio of inner diameter to outer diameter___________________0.5

Flow ratio____0.17

Hydraulic Efficiency_____________________________________95%

Overall Efficiency______________________________________85%

Flow velocity____Constant

Discharge____radial

Neglecting blockage by blades, find the dimensions and blade angles of the turbine.

Solution

Net head, H = 86.4m

Speed of runner, N = 650rpm

Shaft power available, P = 397KW

Ratio of wheel width to wheel diameter at inlet, n = $\frac{B_1}{D_1}$ = 0.1

Ratio of inner diameter to outer diameter = 0.5

Flow ratio, K_f = 0.17

Hydraulic Efficiency, η_h = 95%

Overall Efficiency, η_o = 85%

Flow velocity = Constant i.e. $V_{f1} = V_{f2}$

Discharge is radial i.e. β = 90^0 or V_2 = V_{f2}

Main Dimensions of Turbine:

Flow velocity, V_{f1}= $K_f\sqrt{2gH}$ = 0.17$\sqrt{2\ x\ 9.81\ x85.4}$ = 7m/s

V_{f1} = V_{f2} = 7m/s

Shaft power available from turbine, P = wQH x η_o

397 = 9.81 x Q x 86.4 x 0.85

∴ Discharge Q = $\frac{397}{9.81\ x\ 86.4\ x\ 0.85}$ = 0.551m^3/s

Q = $\pi D_1 B_1 V_{f1}$ neglecting blockage by blades where $D_1\ and\ B_1$ are diameter and width of wheel at inlet respectively.

0.55 = $\pi D_1 x\ 0.1 D_1$ x 7. Recall, B_1= nB_1= nD_1

∴ D_1 = $\left[\frac{0.551}{\pi\ x\ 0.1\ x\ 7}\right]^{1/2}$ = 0.5m

B_1= 0.1D_1 = 0.1 x 0.5 = **0.05m**

Diameter of wheel at outlet, D_2 = 0.5D_1 = 0.5 x 0.5 = **0.25m**

From continuity equation, discharge at inlet = discharge at outlet

∴ Q_1= Q_2

This implies $\pi D_1 B_1 V_{f1}$ = $\pi D_2 B_2 V_{f2}$. Since $V_{f1} = V_{f2}$

$D_1 B_1$= $D_2 B_2$

B_2= $\frac{D_1 B_1}{D_2}$ = $\frac{D_1 B_1}{0.5D_1}$ = 2B_1 = 2 x 0.05 = **0.1m**

Recall, tangential velocity is also known as peripheral velocity at inlet, u_1 = $\frac{\pi D_1 N}{60}$ = $\frac{\pi\ x\ 0.5\ x\ 650}{60}$ = 17m/s

Hydraulic Efficiency, η_h = $\frac{V_{w1} u_1}{gH}$ since V_{w2} = 0 because outlet discharge is radial

0.95 = $\frac{V_{w1}\ x\ 17}{9.81\ x\ 86.4}$

∴ V_{w1} = **47.36m/s**

From inlet triangle, tan α = $\frac{V_{f1}}{V_{w1}}$ = $\frac{7}{47.36}$ = 0.1478

∴ Guide vane angle, α = tan^{-1}0.1478 = **8.4^0**

tanθ = $\frac{V_{f1}}{V_{w1} - u_1}$ = $\frac{7}{47.36 - 17}$ = 0.23

∴ Vane inlet angle, θ= $tan^{-1}0.23$ = **12.95^0**

From the outlet velocity triangle,

u_2 = $\frac{\pi D_2 N}{60}$ = $\frac{\pi\ x\ 0.25\ x\ 650}{60}$ = 8.5m/s

tan$\emptyset$ = $\frac{V_{f2}}{u_2}$ = $\frac{7}{8.5}$ = 0.823

Vane angle at outlet, $\emptyset$ = tan^{-1}0.823 = **39.45^0**

β = 90^0 since discharge is radial at outlet.

Other types of Reaction turbines are:

i. Propeller turbine- Used for heads between 4-80m.
ii. Kaplan turbine- Used when load on turbine remains constant.

The advantages of Kaplan turbine over Francis turbine are:

1. For the same power, Kaplan turbine is more compact in construction and smaller insize.
2. Part-load efficiency is high.
3. Lower frictional losses (because of small number of blades used between 3-8).

Cavitation is the formation, growth and collapse of vapour filled cavities or bubbles in a flowing liquid due to fall in fluid pressure. When the pressure at any point in a flow field equals the vapour pressure of the liquid at that temperature, vapour cavities, also known as bubbles of vapour, begin to appear. Their effects on hydraulic turbines include:

1. Roughening of surface caused by pitting.

2. Vibration
3. Sudden drop in output and efficiency.

Methods to avoid cavitation include:

1. Keep runner/turbine under water or use runner of low specific speed.
2. Select materials which resist better cavitation effect.
3. Polish metal surfaces used and coat them with stainless steel.
4. Use runner of proper specific speed for given head.